



**Shore**

**Year 12  
Term II Examination  
May 2014**

**Mathematics**

**General Instructions**

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11–16, show relevant mathematical reasoning and/or calculations
- Start each of questions 11–16 in a new writing booklet
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and “N/A” on the front cover

Exam Number:
Set:

**Total marks – 100**

**Section I** Pages 2–5

**10 marks**

- Attempt questions 1–10
- Allow about 15 minutes for this section

**Section II** Pages 6–13

**90 marks**

- Attempt questions 11–16
- Allow about 2 hours and 45 minutes for this section

**Note: Any time you have remaining should be spent revising your answers.**

**DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM**

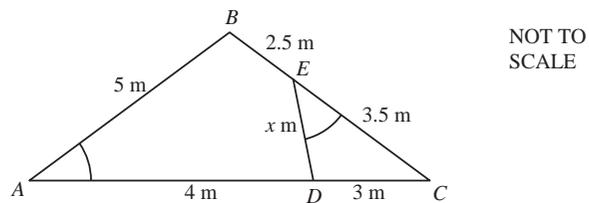
**Section I**

**10 marks**  
**Attempt Questions 1–10**  
**Allow about 15 minutes for this section**

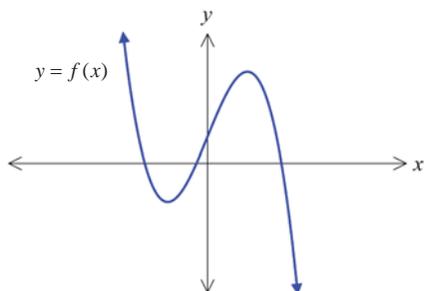
Use the multiple-choice answer sheet for Questions 1–10.

- 
- 1 It is known that for a pair of rational values  $a$  and  $b$ ,  $\frac{2}{2+\sqrt{3}} = a + b\sqrt{3}$ .  
What is the value of  $a$ ?
- (A)  $a = -4$   
(B)  $a = -2$   
(C)  $a = 2$   
(D)  $a = 4$
- 2 For what value of  $x$  does  $a^{2x+1} = \frac{1}{a^3}$ ?
- (A)  $x = -2$   
(B)  $x = -\frac{1}{2}$   
(C)  $x = 1$   
(D)  $x = 2$

- 3 In the diagram below  $\angle CAB = \angle DEC$ . What is the value of  $x$ ?



- (A) 2.5  
 (B) 2.8  
 (C) 3.6  
 (D) 4.375
- 4 Which of the following graphs is a possible gradient function of the function drawn below?



- (A)  $y = f'(x)$
- (B)  $y = f'(x)$
- (C)  $y = f'(x)$
- (D)  $y = f'(x)$

- 5 What is the value of  $\log_{16} 32$ ?

- (A)  $\frac{1}{2}$   
 (B)  $\frac{4}{5}$   
 (C)  $\frac{5}{4}$   
 (D) 2

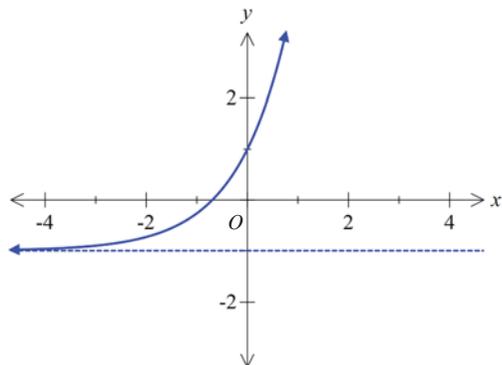
- 6 Which of the following expressions correctly uses Simpson's rule to approximate  $\int_1^5 \frac{dx}{x}$  using 5 function values?

- (A)  $\frac{1}{3} \left[ \left( \frac{1}{1} + \frac{1}{5} \right) + 4 \left( \frac{1}{2} + \frac{1}{4} \right) + 2 \left( \frac{1}{3} \right) \right]$   
 (B)  $\frac{3}{2} \left[ \left( \frac{1}{1} + \frac{1}{5} \right) + 4 \left( \frac{1}{2} + \frac{1}{4} \right) + 2 \left( \frac{1}{3} \right) \right]$   
 (C)  $\frac{3}{2} \left[ \left( \frac{1}{1} + \frac{1}{5} \right) + 2 \left( \frac{1}{2} + \frac{1}{4} \right) + 4 \left( \frac{1}{3} \right) \right]$   
 (D)  $\frac{1}{3} \left[ \left( \frac{1}{1} + \frac{1}{5} \right) + 2 \left( \frac{1}{2} + \frac{1}{4} \right) + 4 \left( \frac{1}{3} \right) \right]$

- 7 What is the primitive of  $3x^2 - \frac{1}{x^2}$ ?

- (A)  $x^3 - x^{-1} + C$   
 (B)  $6x + 2x^{-3} + C$   
 (C)  $x^3 + x^{-1} + C$   
 (D)  $x^3 - \ln x + C$

- 8 Which of the following is a possible equation for the graph drawn below?



- (A)  $y = e^{2x} - 1$   
 (B)  $y = 1 - 2e^x$   
 (C)  $y = 2e^{-x} - 1$   
 (D)  $y = 2e^x - 1$
- 9 Consider geometric series  $1 - 2 + 4 - 8 + 16 - \dots$   
 The sum to  $n$  terms of this series is 2 796 203. For what value of  $n$  is this true?
- (A) 21  
 (B) 22  
 (C) 23  
 (D) no possible value.
- 10 What is the equation of the directrix for the parabola  $(x - 2)^2 = -20(y + 1)$ ?
- (A)  $y = -3$   
 (B)  $y = 4$   
 (C)  $x = 6$   
 (D)  $x = 7$

## Section II

**90 marks**  
**Attempt Questions 11–16**  
**Allow about 2 hours and 45 minutes for this section**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

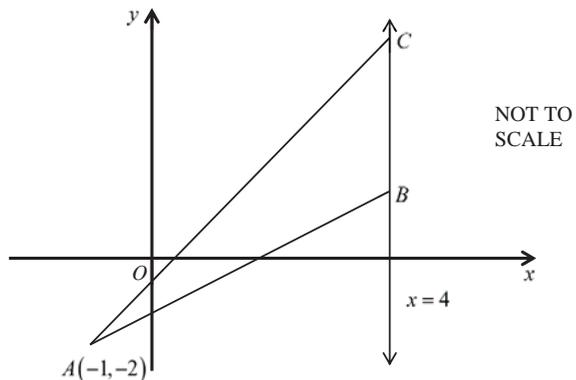
In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

**Question 11** (15 marks) Use a SEPARATE writing booklet.

- (a) Evaluate, correct to 3 significant figures,  $\frac{\sqrt{\pi} - 1}{\sin 30^\circ}$ . 2
- (b) Factorise  $x^2 - 1 - ax - a$ . 2
- (c) Find the gradient of the tangent to the curve  $y = 4 \ln x$  at the point where  $x = e^2$ . 2
- (d) Show that  $f(x) = \frac{x^2 + 2}{x}$  is an odd function. 2
- (e) Find the equation of the line that passes through the point (1, 2) and is perpendicular to  $2x - y + 3 = 0$ . 2
- (f) Find the values of  $x$  for which the geometric series  $(1 - x) + (1 - x)^2 + (1 - x)^3 + \dots$  has a limiting sum? 3
- (g) Find  $\int (e^{5x+4} - 2) dx$ . 2

**Question 12** (15 marks) Use a SEPARATE writing booklet.

- (a) The points,  $A$ ,  $B$  and  $C$  are shown on the number plane below. The point  $A$  has coordinates  $(-1, -2)$ . Points  $B$  and  $C$  lie on the line  $x = 4$ .



- (i) The equation of the line  $AB$  is given as  $3x - 5y - 7 = 0$ .  
Show that the point  $B$  has coordinates  $(4, 1)$ . 1
- (ii) Given the area of  $\triangle ABC$  is  $22.5 \text{ units}^2$ , find the length of  $AC$ . 3
- (b) The quadratic equation  $y = x^2 - \sqrt{2}x - 2$  has the roots  $\alpha$  and  $\beta$ .
- (i) Find  $\alpha + \beta$ . 1
- (ii) Find  $\alpha\beta$ . 1
- (iii) Find  $\alpha^2 + \beta^2$ . 2

**Question 12 continues on page 8**

Question 12 (continued).

- (c) Differentiate  $y = \frac{x^2}{e^{3x}}$  with respect to  $x$ . Write your answer in simplest form. 3
- (d) (i) Differentiate  $y = \sqrt{16 - x^2}$  with respect to  $x$ . 2
- (ii) Hence, or otherwise, find  $\int \frac{2x}{\sqrt{16 - x^2}} dx$ . 2

**End of Question 12**

**Question 13** (15 marks) Use a SEPARATE writing booklet.

- (a) Samantha's parents opened a bank account for her when she was born and deposited \$1 in it. They deposited \$2 on her first birthday, \$4 on her second birthday and so on, with the deposits doubling each birthday. The parents discontinued making deposits after they made a deposit exceeding a million dollars. 3

On what birthday did the deposit exceed a million dollars?

- (b) Find  $\int_0^4 x^3 + \frac{1}{\sqrt{x}} dx$ . 3

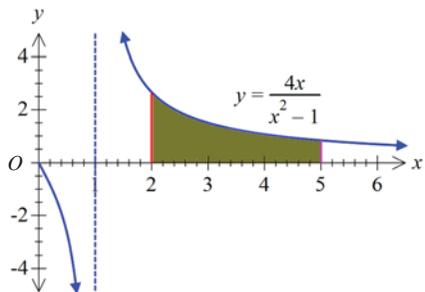
- (c) Evaluate  $\lim_{x \rightarrow 3} \frac{3-x}{x^2-9}$ . 2

- (d) Everyonetown had a population of 50 000 people on the 1<sup>st</sup> of January 2009. The population has grown at a constant rate of 9% every year.

- (i) What was the population of Everyonetown on the 1<sup>st</sup> of January 2014? 2

- (ii) If this population continues to grow at 9% every year, in what year will the population first exceed 100 000 people? 2

- (e) Find the exact value of the area between the curve  $y = \frac{4x}{x^2-1}$ , the  $x$ -axis and the lines  $x = 2$  and  $x = 5$ . 3



**Question 14** (15 marks) Use a SEPARATE writing booklet.

- (a) Consider the parabola  $y^2 - 2y = 12x - 37$ .

- (i) Rewrite this equation in the form  $(y-k)^2 = 4a(x-h)$ . 2

- (ii) Hence sketch the graph of the parabola  $y^2 - 2y = 12x - 37$ , showing the coordinates of the focus. 2

- (b) Consider the function  $f(x) = x(x-3)^2$ .

- (i) Show that  $f'(x) = 3(x-3)(x-1)$ . 1

- (ii) Find all stationary points on  $y = f(x)$  and determine their nature. 3

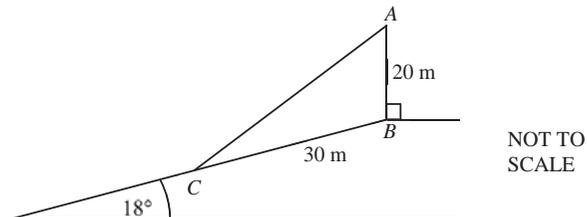
- (iii) Draw a neat sketch of  $y = f(x)$ , showing all intercepts and stationary points. 2

- (c) A function  $f(x)$  has a second derivative  $f''(x) = 36x + 6$ . Find the equation of  $f(x)$  if  $y = f(x)$  has a stationary point at  $(0, -1)$ . 3

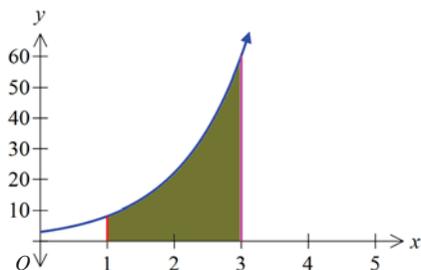
- (d) Solve  $|x+1| = |2x-2|$ . 2

**Question 15** (15 marks) Use a SEPARATE writing booklet.

- (a) A radio mast  $AB$ , of height 20 metres, stands at the top of a slope which is inclined at  $18^\circ$  to the horizontal. The mast is supported by the wire  $AC$  which is attached to the point  $C$  on the slope.  $BC = 30$  metres.

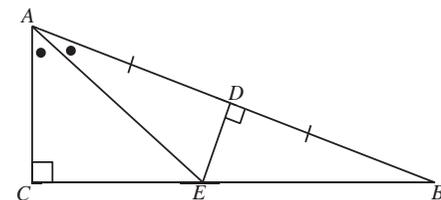


- (i) Calculate the size of  $\angle ABC$ . 1
- (ii) Find the length of the wire  $AC$ . Write your answer correct to the nearest metre. 2
- (b) Show that  $\tan^2 x + 1 + \tan x \sec x = \frac{1 + \sin x}{\cos^2 x}$ . 2
- (c) The area under the curve  $y = 3e^x$  between the values of  $x = 1$  and  $x = 3$  is rotated about the  $x$ -axis. Find the exact volume of the solid of revolution formed. 3



Question 15 continues on page 12

- (d) The diagram below shows a right-angled triangle  $ABC$  with  $\angle ACB = 90^\circ$ . The point  $D$  is the midpoint of  $AB$ , and  $E$  is the point where the perpendicular drawn from  $D$  meets  $BC$ .  $AE$  bisects  $\angle DAC$ .



Copy or trace the diagram into your booklet.

- (i) Prove that  $\triangle ADE \cong \triangle BDE$ . 2
- (ii) Show that  $\angle ABC = 30^\circ$ . 2
- (iii) Hence find the exact ratio  $CB : EB$ . 3

End of Question 15

**Question 16** (15 marks) Use a SEPARATE writing booklet.

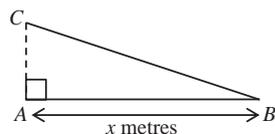
- (a) Stephanie borrowed \$50 000 to start a business. For the first year of the loan Stephanie made no repayments. The loan accrued interest at 9% per annum, compounded monthly. Stephanie then made equal monthly instalments of \$ $M$  for the next 5 years. Let  $A_n$  be the amount owing after  $n$  months.

(i) Show that the amount owing after 15 months is given by 2

$$A_{15} = 50\,000(1.0075)^{15} - M(1 + 1.0075 + 1.0075^2).$$

- (ii) Calculate Stephanie's monthly repayment of \$ $M$  correct to the nearest cent. 3

- (b) A wire of length 10 metres is bent to form the hypotenuse and base of a right angled triangle  $ABC$ , as shown in the diagram below. Let the length of the base  $AB$  be  $x$  metres.



NOT  
TO SCALE

(i) Show that the area of the triangle  $ABC$  in square metres is given by 2

$$A = \frac{x}{2} \sqrt{100 - 20x}.$$

- (ii) Find the value of  $x$  that gives the greatest possible area. 3

- (c) Consider the graphs  $y = \sqrt{x}$  and  $y = mx$ , where  $m$  is a constant such that  $m > 0$ .

(i) Show that the two graphs intersect at  $(0, 0)$  and  $\left(\frac{1}{m^2}, \frac{1}{m}\right)$ . 2

- (ii) Hence find the area enclosed by the two graphs in terms of  $m$ . 3

**End of Paper**

1) D

$$\frac{2}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2(2-\sqrt{3})}{4-3}$$

$$= 4 - 2\sqrt{3}$$

$\therefore a = 4, b = -2$

2) A

$$a^{2x+1} = \frac{1}{a^2}$$

$$a^{2x+1} = a^{-3}$$

$$2x+1 = -3$$

$$2x = -4$$

$$x = -2$$

3) A

$$\frac{x}{5} = \frac{3}{6}$$

$$x = 2.5$$

4) B

5) C

$$2^5 = 2^{4x} \quad \log_{16} 32 = x$$

$$5 = 4x \quad \text{or} \quad 32 = 16^x$$

$$x = \frac{5}{4} \quad \ln(32) = x \ln(16)$$

$$x = \frac{\ln(32)}{\ln(16)}$$

$$x = \frac{5}{4}$$

6) A

7) C

$$f(x) = 3x^2 - \frac{1}{x^2}$$

$$= 3x^2 - x^{-2}$$

$$\int f(x) dx = \int 3x^2 - x^{-2} dx$$

$$= x^3 + x^{-1} + C$$

d)

$$f(x) = \frac{x^2 + 2}{x}$$

$$f(-x) = \frac{(-x)^2 + 2}{-x}$$

$$= -\frac{x^2 + 2}{x}$$

$$= -f(x)$$

e)

$$2x - y + 3 = 0$$

$$y = 2x + 3$$

$$m_1 = 2$$

$$m_2 = -\frac{1}{2}, \text{ through } (1, 2)$$

$$y - 2 = -\frac{1}{2}(x - 1)$$

$$2y - 4 = -x + 1$$

$$x + 2y - 5 = 0$$

f)

$$(1-x) + (1-x)^2 + (1-x)^3 + \dots$$

$$r = (1-x)$$

for limiting sum

$$-1 < r < 1$$

$$-1 < 1-x < 1$$

$$-2 < -x < 0$$

$$0 < x < 2$$

g)

$$\int e^{(5x+4)} - 2 dx = \frac{1}{5} e^{(5x+4)} - 2x + C$$

8) D

9) C

$$1 - 2 + 4 - 8 + 16 - \dots$$

$$a = 1, r = -2, S_n = 2796203$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$2796203 = \frac{1((-2)^n - 1)}{-2 - 1}$$

$$-8388609 = (-2)^n - 1$$

$$-8388608 = (-2)^n$$

$$(-1)8388608 = (-1)^n (2)^n$$

$$n = \frac{\ln(8388608)}{\ln(2)} \text{ for } n \text{ odd}$$

$$= 23$$

10) B

11)

a)  $\frac{\sqrt{\pi} - 1}{\sin 30^\circ} = 1.544907702\dots$

$$\approx 1.54$$

b)  $x^2 - 1 - ax - a = (x-1)(x+1) - a(x+1)$

$$= (x+1)(x-1-a)$$

c)  $y = 4 \ln x$

$$\frac{dy}{dx} = \frac{4}{x}$$

at  $x = e^2$

$$\frac{dy}{dx} = \frac{4}{e^2}$$

12)

a)

i)  $3x - 5y - 7 = 0$

at  $x = 4$

$$3(4) - 5y - 7 = 0$$

$$12 - 7 = 5y$$

$$y = 1$$

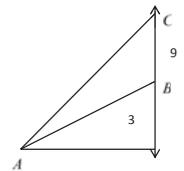
ii) Area of  $\triangle ABC = 22.5$

$$22.5 = \frac{1}{2}(5)BC$$

$$BC = 9$$

$$AC = \sqrt{5^2 + (9+3)^2}$$

$$= 13 \text{ units}$$



b)

i)  $y = x^2 - \sqrt{2}x - 2$

$$\alpha + \beta = \frac{-b}{a}$$

$$= \frac{-(-\sqrt{2})}{1}$$

$$= \sqrt{2}$$

ii)  $\alpha\beta = \frac{c}{a}$

$$= \frac{-2}{1}$$

$$= -2$$

iii)  $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= (\sqrt{2})^2 - 2(-2)$$

$$= 2 + 4$$

$$= 6$$

c)

$$y = \frac{x^2}{e^{3x}}$$

$$\frac{dy}{dx} = \frac{e^{3x} 2x - x^2 3e^{3x}}{e^{6x}}$$

$$= \frac{xe^{3x}(2-3x)}{e^{6x}}$$

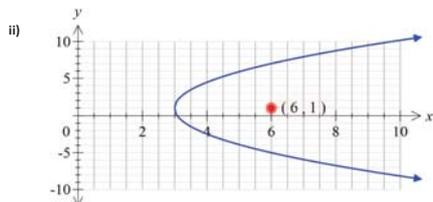
$$= \frac{x(2-3x)}{e^{3x}}$$

d) i)  $y = \sqrt{16-x^2}$   
 $= (16-x^2)^{\frac{1}{2}}$   
 $\frac{dy}{dx} = \frac{1}{2} \times (16-x^2)^{-\frac{1}{2}} \times -2x$   
 $= \frac{-x}{\sqrt{16-x^2}}$

ii)  $\int \frac{2x}{\sqrt{16-x^2}} dx$   
 $= -2 \int \frac{-x}{\sqrt{16-x^2}} dx$   
 $= -2\sqrt{16-x^2} + C$

13) a)  $T_n = ar^{n-1}$   
 $1000000 = 1 \times 2^{n-1}$   
 $1000000 = \frac{2^n}{2}$   
 $2000000 = 2^n$   
 $n = \frac{\ln(2000000)}{\ln(2)}$   
 $= 20.931\dots$   
 $\therefore$  last deposit on her 20th Birthday

b)  $\int_0^4 x^3 + \frac{1}{\sqrt{x}} dx$   
 $= \left[ \frac{x^4}{4} + 2x^{\frac{1}{2}} \right]_0^4$   
 $= \left[ \frac{(4)^4}{4} + 2(4)^{\frac{1}{2}} \right] - \left[ \frac{(0)^4}{4} + 2(0)^{\frac{1}{2}} \right]$   
 $= (64+4) - 0$   
 $= 68$



b) i)  $f(x) = x(x-3)^2$   
 $f'(x) = 2x(x-3) + (x-3)^2$   
 $= (x-3)[2x+x-3]$   
 $= (x-3)(3x-3)$   
 $= 3(x-3)(x-1)$

ii)  $f'(x) = 3(x-3)(x-1)$   
 $= 3x^2 - 12x + 9$   
 Stationary points at  $f'(x) = 0$   
 $x = 3$  and  $x = 1$   
 $f(1) = 1(1-3)^2$   
 $= 4$   
 $f(3) = 3(3-3)^2$   
 $= 0$   
 $f''(x) = 6x^2 - 12$   
 $f''(1) = 6(1)^2 - 12$   
 $= -6$   
 $< 0$   
 $\therefore$  Maximum Turning point at (1, 4)

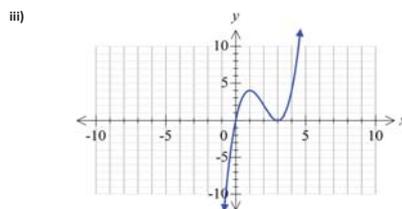
$f''(x) = 6x^2 - 12$   
 $f''(3) = 6(3)^2 - 12$   
 $= 42$   
 $> 0$   
 $\therefore$  Minimum Turning point at (3, 0)

c)  $\lim_{x \rightarrow 3} \frac{3-x}{x^2-9} = \lim_{x \rightarrow 3} \frac{-(x-3)}{(x+3)(x-3)}$   
 $= \lim_{x \rightarrow 3} \frac{-1}{x+3}$   
 $= \frac{-1}{(3+3)}$   
 $= -\frac{1}{6}$

d) i)  $P = 50000(1+0.09)^5$   
 $= 76931.19775$   
 $\approx 76931$   
 ii)  $100000 = 50000(1.09)^n$   
 $n = \frac{\ln(2)}{\ln(1.09)}$   
 $= 8.043231\dots$   
 The population will first exceed 100000 in 2017

e)  $A = \int_2^5 \frac{4x}{x^2-1} dx$   
 $= 2 \int_2^5 \frac{2x}{x^2-1} dx$   
 $= 2 \left[ \ln(x^2-1) \right]_2^5$   
 $= 2[\ln(24) - \ln(3)]$   
 $= 2 \ln 8$   
 $= 6 \ln 2$  units<sup>2</sup>

14) a)  $y^2 - 2y = 12x - 37$   
 i)  $y^2 - 2y + 1 = 12x - 37 + 1$   
 $(y-1)^2 = 12x - 36$   
 $(y-1)^2 = 12(x-3)$



c)  $f''(x) = 36x + 6$   
 $f'(x) = \int f''(x) dx$   
 $= 18x^2 + 6x + C_1$   
 $f'(0) = 0$   
 $0 = 18(0)^2 + 6(0) + C_1$   
 $C_1 = 0$   
 $f'(x) = 18x^2 + 6x$   
 $f(x) = \int f'(x) dx$   
 $= 6x^3 + 3x^2 + C_2$   
 $f(0) = -1$   
 $-1 = 6(0)^3 + 3(0)^2 + C_2$   
 $C_2 = -1$   
 $\therefore f(x) = 6x^3 + 3x^2 - 1$

d)  $|x+1| = |2x-2|$   
 case 1  
 $x+1 = 2x-2$   
 $x = 3$   
 case 2  
 $x+1 = -2x+2$   
 $x = \frac{1}{3}$   
 test  $x = \frac{1}{3}$   
 $\therefore x = 3, x = \frac{1}{3}$

15)

a)

i)  $\angle ABC = 90^\circ + 18^\circ$   
 $= 108^\circ$

ii)  $AC^2 = (30)^2 + (20)^2 - 2 \times 30 \times 20 \cos 108^\circ$   
 $= 1670.820393\dots$   
 $AC = 40.87566\dots$   
 $\approx 41 \text{ m}$

b)

$$\tan^2 x + 1 + \tan x \sec x = \frac{1 + \sin x}{\cos^2 x}$$

$$LHS = \tan^2 x + 1 + \tan x \sec x$$

$$= \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} + \frac{\sin x}{\cos x} \times \frac{1}{\cos x}$$

$$= \frac{1 + \sin x}{\cos^2 x}$$

$$= RHS$$

c)

$$y = 3e^x$$

$$y^2 = (3e^x)^2$$

$$= 9e^{2x}$$

$$V = \pi \int_1^3 9e^{2x} dx$$

$$= \pi \left[ \frac{9e^{2x}}{2} \right]_1^3$$

$$= \pi \left[ \frac{9e^{2(3)}}{2} - \frac{9e^{2(1)}}{2} \right]$$

$$= \frac{9\pi(e^6 - e^2)}{2} \text{ units}^2$$

$$= \frac{9\pi e^2(e^4 - 1)}{2} \text{ units}^2$$

d)

i)  $\triangle ADE \cong \triangle BDE$   
 $AD = BD$  (Given)  
 $DE = DE$  (Common)  
 $\angle ADE = \angle BDE = 90^\circ$   
 $\therefore \triangle ADE \cong \triangle BDE$  (SAS)

ii)  $\angle CAE = \angle EAD$  (AE bisects  $\angle BAC$ )

let  $\angle CAE = \alpha$   
 $\angle EAD = \angle EBD$  (matching angles in congruent triangles)  
 $3\alpha = 90^\circ$  (Angle sum of a triangle ABC)  
 $\alpha = 30^\circ$   
 $\angle ABC = 30^\circ$

iii) let  $AB = 2$

$$\sin 30^\circ = \frac{AC}{2}$$

$$AC = 1$$

$$CB = \sqrt{2^2 - 1^2}$$

$$= \sqrt{3}$$

$$DB = 1$$

$$\cos 30^\circ = \frac{EB}{AB}$$

$$EB = \frac{2}{\sqrt{3}}$$

$$CB : EB$$

$$\sqrt{3} : \frac{2}{\sqrt{3}}$$

$$3 : 2$$

16)

a)

i)  $A_1 = 50000(1.0075)^1$   
 $A_2 = 50000(1.0075)^2$   
 $A_3 = 50000(1.0075)^3 - M$   
 $A_4 = [50000(1.0075)^3 - M](1.0075) - M$   
 $A_4 = 50000(1.0075)^4 - M(1 + 1.0075)$   
 $A_5 = 50000(1.0075)^5 - M(1 + 1.0075 + 1.0075^2)$

ii)  $A_{72} = 50000(1.0075)^{72} - M(1 + 1.0075 + \dots + 1.0075^{69})$   
 $A_{72} = 0$   
 $M = \frac{50000(1.0075)^{72}}{1 + 1.0075 + \dots + 1.0075^{69}}$   
 $1 + 1.0075 + \dots + 1.0075^{69} = \frac{1(1.0075^{70} - 1)}{0.0075}$   
 $M = \frac{50000(1.0075)^{72}}{1.0075^{70} - 1} \times 0.0075$   
 $M = \frac{375(1.0075)^{72}}{1.0075^{70} - 1}$   
 $= \$1135.28$

b)

i)  $AC = \sqrt{(10-x)^2 - x^2}$   
 $= \sqrt{100 - 20x + x^2 - x^2}$   
 $= \sqrt{100 - 20x}$   
 $Area_{\triangle ABC} = \frac{1}{2} x \sqrt{100 - 20x}$   
 $= \frac{x}{2} \sqrt{100 - 20x}$

ii)  $A = \frac{x}{2} \sqrt{100 - 20x}$   
 $= \frac{x(100 - 20x)^{\frac{1}{2}}}{2}$   
 $\frac{dA}{dx} = \frac{-10x}{\sqrt{100 - 20x}} + \frac{\sqrt{100 - 20x}}{2}$   
 $= \frac{-10x + (100 - 20x)}{2\sqrt{100 - 20x}}$

Stat points at  $\frac{dA}{dx} = 0$

$$0 = \frac{-10x + (100 - 20x)}{2\sqrt{100 - 20x}}$$

$$0 = 100 - 30x$$

$$x = \frac{10}{3} m$$

x	3	$\frac{10}{3}$	4
$\frac{dA}{dx}$	$\frac{-10(3) + (100 - 20(3))}{2\sqrt{100 - 20(3)}}$	0	$\frac{-10(4) + (100 - 20(4))}{2\sqrt{100 - 20(4)}}$
	$= \frac{-30 + 40}{2\sqrt{40}}$		$= \frac{-40 + 20}{2\sqrt{40}}$
	$> 0$		$< 0$
			

Therefore max area when  $x = \frac{10}{3} m$ .

c)

i)

$$y = \sqrt{x}$$

$$y = mx$$

$$\sqrt{x} = mx$$

$$0 = \sqrt{x} - mx$$

$$= x \left( x^{-\frac{1}{2}} - m \right)$$

$$x = 0$$

$$\therefore y = 0$$

and

$$x^{-\frac{1}{2}} = m$$

$$x = m^{-2}$$

$$= \frac{1}{m^2}$$

$$\therefore y = \sqrt{\frac{1}{m^2}}$$

$$= \frac{1}{m}$$

ii)  $\therefore$  The graphs intersect at  $(0,0)$  and  $\left(\frac{1}{m^2}, \frac{1}{m}\right)$ .

$$A = \int_0^{\frac{1}{m^2}} (\sqrt{x} - mx) dx$$

$$= \left[ \frac{2x^{\frac{3}{2}}}{3} - \frac{mx^2}{2} \right]_0^{\frac{1}{m^2}}$$

$$= \left[ \frac{2\left(\frac{1}{m^2}\right)^{\frac{3}{2}}}{3} - \frac{m\left(\frac{1}{m^2}\right)^2}{2} \right] - 0$$

$$= \frac{2\left(\frac{1}{m^3}\right)}{3} - \frac{\left(\frac{1}{m^3}\right)}{2}$$

$$= \left(\frac{2}{3m^3}\right) - \left(\frac{1}{2m^3}\right)$$

$$= \frac{1}{6m^3} \text{ units}^2$$